heat fluxes; s, area of contact; T, temperature; u, u_0, filtration speed and fluidization onset speed; x_i, stress axes; α, proportion of conducting particles in binary mixture; $\beta(\mu)$, ratio of effective conductivity of medium containing noncontacting particles to the effective conductivity of the continuous phase for $\lambda_1 \gg \lambda_0$; γ, particle density minus specific buoyancy; δ, compression length; ζ, coordination number; θ, φ, angular coordinates of contact relative to mean flow direction; $\varkappa$, exponent in (13) and (14); Λ_i, eigenvalues of conductivity tensor Λ; λ_0, λ_1, conductivities of continuous and dispersed phases; ν, fraction of surface area represented by a single contact; ρ, volume content of dispersed phase; σ, compressive stress; τ, mean temperature; Φ, distribution function; $\psi(u)$, hydraulic force per unit particle volume; *, values referred to one particle; $\langle\ \rangle$, averages.

LITERATURE CITED

1. A. F. Chudnovskii, Thermophysical Characteristics of Dispersed Materials [in Russian], Fizmatgiz, Moscow (1962).
2. G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composites [in Russian], Énergiya, Leningrad (1974).
3. Yu. A. Buevich and Yu. A. Korneev, Inzh.-Fiz. Zh., 31, No. 4 (1976).
4. D. L. Swift, Intern. J. Heat Mass Transf., 9, 1061 (1966).
5. V. A. Borodulya, High-Temperature Processes in an Electrothermal Fluidized Bed [in Russian], Nauka i Tekhnika, Minsk (1973).
6. R. G. Deissler and J. S. Boegli, Trans. ASME, 80, 1417 (1958).
7. N. Wakao and D. Vortmeyer, Chem. Eng. Sci., 26, 1753 (1971).
8. L. D. Landau and E. M. Lifshits, Theory of Elasticity, 2nd ed., Addison-Wesley (1971).
9. M. É. Azrov and O. M. Todes, The Hydraulic and Thermal Principles of Equipments with Stationary Fluidized Beds [in Russian], Khimiya, Leningrad (1968).
10. A. K. Reed and W. M. Goldberger, Chem. Eng. Progr. Symp., Ser. 62, No. 7 (1966).
11. S. Kirkpatrick, Rev. Mod. Phys, 45, 574 (1973).
12. B. I. Shklovskii and A. L. Éfros, Usp. Fiz. Nauk, 117, 401 (1975).

A COMBINED NUMERICAL METHOD FOR DETERMINING THE CONDUCTANCE OF COMPOSITE BODIES

G. N. Dul'nev, M. A. Eremeev, Yu. P. Zarichnyak, and E. N. Koltunova

UDC 536.242: 518.61

We propose a new numerical method (a combination of the method of grids with Rayleigh's method) which is very promising for the calculation of potential fields, fluxes, and conductance of composite bodies, especially in the case of components with sharply differing properties.

We consider a two-component region in the form of a cylinder made up of two hemispheres which are in contact at the point A (Fig. 1). As an example, we consider the problem of determining the effective conductance, say the effective thermal conductivity, of the composite region. We denote the thermal conductivity of the material of the hemispheres by λ_1 and that of the material filling the gap between them by λ_2, where λ_1 and λ_2 may be substantially different. Suppose (for the sake of definiteness) that the bases of the cylinder are isopotential (isothermal) planes and that the lateral surface is impenetrable to the streamlines (an adiabatic surface). Such a composite system is often used for constructing a model of the structure of granular materials when we calculate their effective coefficients of generalized conductance (thermal conductivity, electrical conductivity, dielectric permittivity, magnetic permeability, etc.).

Leningrad Institute of Precision Mechanics and Optics.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 2, pp. 284-291, February, 1977. Original article submitted February 9, 1976.

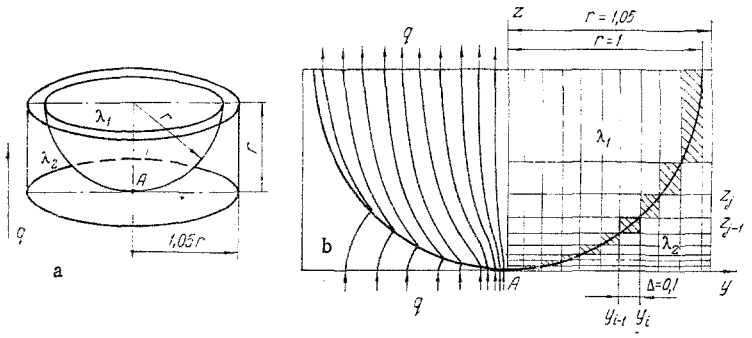

Fig. 1. Model of structural granular materials: a) axonometric representation of model; b) distribution of streamlines in model and subdivision into blocks by a grid.

Since we have no exact solution of the problem for the conductance of the model for an arbitrary relationship between the properties of the components λ_1 and λ_2, it becomes necessary to use various approximate methods for the analytical determination of the conductance. The error in the approximate methods can be estimated and the boundaries of their applicability can be determined by comparing the result of an approximate calculation with an analogous quantity obtained by a numerical method which, for sufficiently small error, can be taken as the standard.

In order to carry out the numerical calculation, we subdivide the composite region into small segments (blocks). If the properties of the components differ greatly (for example, $\lambda_2 \ll \lambda_1$), the concentration of streamlines (the streamline density) (Fig. 1b) in the region near the point of contact is much greater than at the periphery, so that we must use a grid with a nonuniform step which decreases as we approach the point of contact. We therefore subdivide the region in such a way that the dimensions of the grid step along the z axis will be determined by the intersection of the grid along the y axis with the sphere—gap component interface. We assign the subscripts i and j to the coordinates of the grid lines (Fig. 1b).

As a result, the composite region is subdivided into blocks most of which are filled with homogeneous components (with thermal conductivity λ_1 or λ_2), while a smaller number of them (the blocks intersected by the component interface) are filled with components of different kinds.

The proposed method of subdividing along the z axis considerably simplifies our further description, since it enables us to have only one type of two-component blocks with a spherical interface passing between diagonally opposite vertices (Fig. 2) instead of the five types of two-component blocks obtained in the investigation of a uniform subdivision along the y and z axes in [1]. The bases of a two-component block will also be regarded as plane isotherms, while the lateral surface parallel to the z axis will again be taken to be adiabatic.

Conductance of the Blocks

The determination of the conductance of a small two-component block is an independent problem, which we shall solve by Rayleigh's method [2].

First we divide a two-component block by means of isothermal planes parallel to the bases into infinitesimally thin flat layers of thickness dz (Fig. 2a). The thermal conductance of such a flat two-component layer in the z direction will be denoted by $d\sigma_{sz}$ (where s is the subscript for the isothermal subdivision) and calculated as the conductance of two flat walls of thickness dz with thermal conductivities λ_1 and λ_2 which are positioned perpendicular to the heat flux.

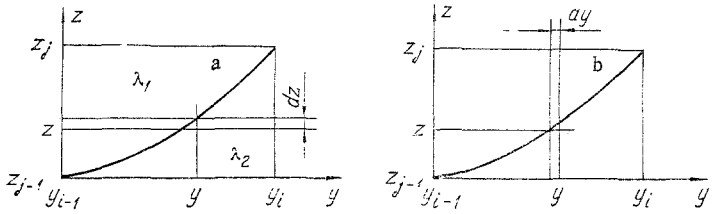

Fig. 2. Example of the separation of a two-component block by isothermal (a) and adiabatic (b) surfaces.

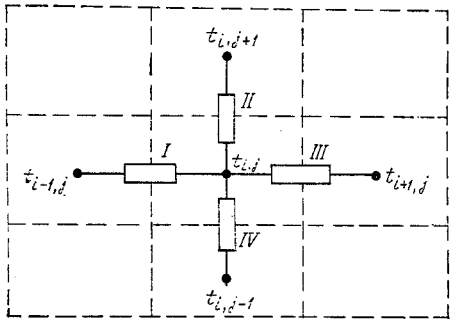

Fig. 3. Scheme of interaction of blocks in the calculation of the temperature t_{ij} in an individual block ij: I) $\sigma_{y|(i-1)-i,j|} = \frac{1}{2}[\sigma^{-1}_{y|i,j|} + \sigma^{-1}_{y(i-1,j}]^{-1}$; II) $\sigma_{z|i,(j+1)-j|} = \frac{1}{2}[\sigma^{-1}_{z|i,j|} + \sigma^{-1}_{z|i,j+1|}]^{-1}$; III) $\sigma_{y|i-(i+1),j|} = \frac{1}{2}[\sigma^{-1}_{y|i,j|} + \sigma^{-1}_{y|i+1,j|}]^{-1}$; IV) $\sigma_{z|i,j-(j-1)|} = \frac{1}{2}[\sigma^{-1}_{z|i,j|} + \sigma^{-1}_{z|i,j-1|}]^{-1}$.

The thermal conductance σ_{szij} of a two-component block in the z direction can be obtained by summing (integrating) the thermal resistances of the system of successively positioned layers of thickness dz, i. e.,

$$\sigma_{szij} = \frac{1}{R_{szij}} = \frac{1}{\int_{z_{j-1}}^{z_j} dR_{szij}}. \tag{1}$$

Each elementary resistance dR_{szij} is formed by the parallel combination of the resistance dR'_{szij} and dR''_{szij} (Fig. 2a) of a particle and the void-filling component; therefore,

$$dR_{szij} = [(dR'_{szij})^{-1} + (dR''_{szij})^{-1}]^{-1}, \tag{2}$$

where

$$dR'_{szij} = \frac{dz}{\pi\lambda_1 (y^2 - y^2_{i-1})}, \quad dR''_{szij} = \frac{dz}{\pi\lambda_2 (y_i^2 - y^2)}.$$

At the component interface the grid coordinates (for i = j) y_i and z_j are related by the equation $y_i^2 = 1-(1-z_j)^2$. Therefore,

$$dR'_{szij} = \frac{dz}{\pi\lambda_1 [(1-z_{j-1})^2 - (1-z)^2]}, \qquad dR''_{szij} = \frac{dz}{\pi\lambda_2 [(1-z)^2 - (1-z_j)^2]}. \tag{3}$$

From the expressions (2) and (3)

$$dR_{szij} = \frac{dz}{\pi\lambda_1 [(1-z_{j-1})^2 - (1-z_j)^2 - (1-\nu)(1-z)^2]}, \quad \nu = \frac{\lambda_2}{\lambda_1}. \tag{4}$$

Substituting (4) into (1) and carrying out the integration, we obtain an expression for calculating the conductance of block ij in the z direction:

$$\sigma_{szij} = \frac{2\pi\sqrt{1-\nu}\sqrt{(1-\nu) - (y_{i-1} - \nu y_i^2)}}{\ln \dfrac{\sqrt{(1-\nu)(1-y^2_{i-1})} + \sqrt{(1-\nu) - (y^2_{i-1} - \nu y_i^2)}}{\sqrt{(1-\nu)(1-y_i^2)} + \sqrt{(1-\nu) - (y^2_{i-1} - \nu y_i^2)}}}. \tag{5}$$

It is known that the conductance value calculated from expression (1) will be systematically higher than its true value σ_z^{tr}, i. e., $\sigma_{szij} = \sigma_z^{max} > \sigma_z^{tr}$. In the y direction the conductance σ_{syij} can be calculated in an analogous manner, but unfortunately the integration for obtaining the calculated expression can be carried out only by numerical methods.

We therefore simplify the problem and define σ_{syij} as the conductance of a block completely filled with the material of a particle:

$$\sigma_{syij} = 2\pi\lambda_1 \frac{\sqrt{1-y^2_{i-1}} - \sqrt{1-y_i^2}}{\ln \dfrac{y_i}{y_{i-1}}}. \tag{6}$$

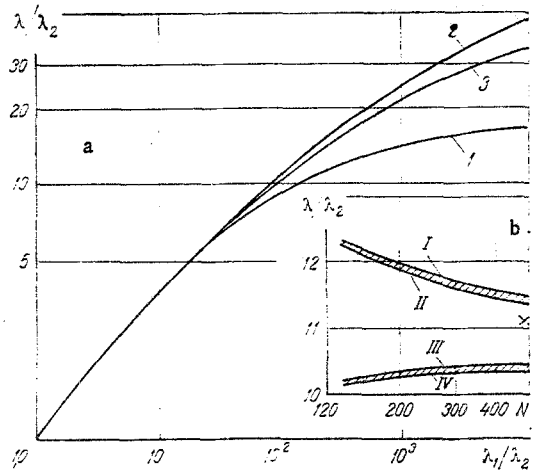

Fig. 4. Comparison of the results of calculating the effective thermal conductivity of the model as a function: a) of the ratio of thermal conductivities of the components [1) Vakao and Vortmeyer [1]; 2, 3) λ^{max}/λ_2 and λ^{min}/λ_2 — the method proposed here]; b) of the number of blocks N in the grid, $\nu = 0.01$ [I) $\lambda^{max}_{out}/\lambda_2$; II) $\lambda^{max}_{in}/\lambda_2$; III) $\lambda^{min}_{out}/\lambda_2$; IV) $\lambda^{min}_{in}/\lambda_2$; the marked point is for λ^{max}/λ_2].

Obviously, the value calculated by (6) for the conductance σ_{syij} will be too high, and therefore we can use it for calculating λ^{max}.

Now we shall subdivide the blocks into infinitesimally thin layers of width dy by using a system of adiabatic surfaces perpendicular to the isothermal bases of the two-component block (Fig. 2b). The thermal resistance of each thin layer dR_{ay} can be represented as the sum of the thermal resistances of two flat walls of width dy filled with components having thermal conductivities λ_1 and λ_2.

The thermal conductance of the block in the direction of the z axis calculated by this method will be denoted by σ_{azij}; we can obtain it from the expression

$$\sigma_{azij} = \int_{y_{i-1}}^{y_i} d\sigma_{azij}, \quad d\sigma_{azij} = [(d\sigma'_{azij})^{-1} + (d\sigma''_{azij})^{-1}]^{-1}, \tag{7}$$

where $d\sigma'_{azij}$ and $d\sigma''_{azij}$ are the conductance values of the segments of an annular layer with thermal conductivities λ_1 and λ_2;

$$d\sigma'_{azij} = \frac{2\pi\lambda_1 z dz}{\sqrt{1-y^2} - \sqrt{1-y_i^2}}, \quad d\sigma''_{azij} = \frac{2\pi\lambda_2 z dz}{\sqrt{1-y_{i-1}^2} - \sqrt{1-y^2}}. \tag{8}$$

Substituting $d\sigma'_{azij}$ and $d\sigma''_{azij}$ from (8) into (7) and carrying out the integration, we find the required conductance

$$\sigma_{azij} = \frac{2\pi\nu}{1-\nu}\left[\left(1 + \frac{\sqrt{1-y_{i-1}^2} - \sqrt{1-y_i^2}}{1-\nu}\right)\ln\frac{1}{\nu} - (\sqrt{1-y_{i-1}^2} - \sqrt{1-y_i^2})\right]. \tag{9}$$

It is known that the conductance value σ_{azij} determined from the expression (7) will be systematically lower than its true value, $\sigma_{azij} = \sigma_z^{min} < \sigma_z^{tr}$.

The conductance in the y direction will be defined, for the sake of simplicity, as the conductance of a block completely filled with the void-filling component, i. e.,

$$\sigma_{ayij} = 2\pi\lambda_2 \frac{\sqrt{1-y_{i-1}^2}-\sqrt{1-y_i^2}}{\ln\frac{y_i}{y_{i-1}}}. \tag{10}$$

The conductance of the homogeneous blocks can be calculated by the well-known simple formulas for a plane wall in the z direction and for a cylindrical wall in the y direction. The scheme of interaction for calculating the temperature t_{ij} in a specific block ij is shown in Fig. 3.

Knowing the conductance of the individual blocks $\sigma_{z,y}{}^{\max,\min}_{ij}$, we can use a relaxation method for calculating the values of the temperatures at the nodes as weighted average values of the temperatures of the adjacent blocks:

$$t_{ij} = \frac{t_{i+1,\,j}\sigma_{y|i\;\;(i+1),j|} + t_{i-1,j}\sigma_{y|(i-1)\;\;i,\,j|}}{\sigma_{y|i\;\;(i+1),j|} + \sigma_{y|(i-1)\;\;i,\,j|}} + \frac{t_{i,\,j+1}\sigma_{z|i,\,(j+1)\;\;j|} + t_{i,\,j-1}\sigma_{z|i,\,j\;\;(j-1)|}}{+\,\sigma_{z|i,\,(j+1)\;\;j|} + \sigma_{z|i,\,j\;\;(j-1)|}}.$$

The error in calculating the temperature field in a radial element under consideration (Fig. 1a) can be estimated by comparing the values of the heat flux q_{in} at the inlet (the lower base of the radial element) and q_{out} at the outlet (the upper base).

Using the values σ_{szij}, σ_{syij} for the two-component blocks, we can obtain higher-than-true values for the fluxes q_{in}^{max} and q_{out}^{max}:

$$q_{in}^{max,min} = \sum_{i=1}^{n}(t_{in} - t_{ij})\,2\sigma_{szij|i,\,j=1}^{max,min},$$
$$q_{out}^{max,min} = \sum_{i=1}^{n}(t_{i,j=n} - t_{out})\,2\sigma_{szij|i,\,j=n}^{max,min}. \tag{11}$$

The effective conductivity λ of the radial element (and of the model as a whole) in the z direction is calculated on the basis of Fourier's law, assuming that the entire volume is filled with a substance having conductivity λ:

$$\lambda^{max,min} = \frac{q^{max,min}}{t_{in} - t_{out}} \cdot \frac{H}{S}, \tag{12}$$

where H and S are the height and area of the radial element or of the model as a whole.

In Fig. 4b we have constructed the graph of the ratio of the effective thermal conductivity to the thermal conductivity of the second component as a function of the number of blocks into which the composite body has been subdivided. The body was subdivided along the y axis into blocks with a uniform step, and the ratio of thermal conductivities of the component parts of the body was $\nu = 0.01$. From Fig. 4b it can be seen that the relative deviation

$$\frac{\Delta q}{q} = \frac{(q_{in} - q_{out})^{max,min}}{0.5(q_{in} + q_{out})^{max,min}} \cdot 100\% \tag{13}$$

does not exceed 1% and that as the number of blocks increases, the value of λ^{max} decreases, while λ^{min} increases; consequently, the true value of the effective conductivity λ will lie in the interval between λ^{max} and λ^{min} and will differ from the arithmetic mean value $\lambda = 0.5(\lambda^{max} + \lambda^{min})$ by no more than the quantity $\Delta\lambda = 0.5(\lambda^{max} - \lambda^{min})$.

Results of the Calculations and Estimate of the Error

It can be seen from Fig. 4b that with a grid having a uniform step along the y axis, when we used 506 blocks, the relative deviation $\Delta\lambda/\bar{\lambda}$ between the maximum and minimum values of the effective conductivity was about 10% for $\nu = 0.01$, and we do not expect the values of λ^{max} and λ^{min} to get substantially closer to each other. The average amount of machine time used for calculating one point on the Minsk-22 electronic computer was 35 min.

For a greater difference between the properties of the components (for example, when particles with high thermal conductivity are used in combination with air, we have $1 \cdot 10^2 < \lambda_1/\lambda_2 < 1 \cdot 10^4$), the difference $\Delta\lambda/\bar{\lambda}$ exceeded 100% and made it impossible to use the $\bar{\lambda}$ as standard values for estimating the errors of the approximate methods for calculating the effective thermal conductivity.

Therefore, when $\lambda_1/\lambda_2 \geq 1 \cdot 10^3$, it became advisable to use a nonuniform subdivision along the y axis, with a step of

$$y_i - y_{i-1} = \left(\frac{i}{n}\right)^2 y_n \tag{14}$$

(in Fig. 4b this is indicated by a point), or even

$$y_i - y_{i-1} = \left(\frac{i}{n}\right)^3 y_n.$$

In Fig. 4a we show the values of the effective thermal conductivity which were calculated by the proposed method. As can be seen from the figure, the difference between the values of λ^{max} and λ^{min} (curves 2 and 3) becomes considerably larger as the ratio λ_1/λ_2 increases. However, even for $\lambda_1/\lambda_2 = 1 \cdot 10^4$ the λ^{max} and λ^{min} values calculated by the proposed method differ by no more than 30%, and their arithmetic mean value differs from the true value by less than 15% and may be regarded as a standard for estimating the error of other approximate methods.

Comparing the calculated functions for the effective thermal conductivity which were obtained by Vakao and Vortmeyer [1] (curve 1 of Fig. 4a) and by us, we note that the Vakao—Vortmeyer values are considerably lower (for $\lambda_1/\lambda_2 > 10^3$ the difference amounts to 100%). The reason for this difference, in our opinion, is that the conductances of the boundary blocks were too crudely calculated and that the region was subdivided uniformly.

To obtain more precise "standard" values, we would have to make the grid denser and increase the computation time.

By using this combination of the grid method and the Rayleigh method, we can eliminate the systematic errors due to the fact that the model represents the component interface in two-component blocks; it gives us very useful results in the case when the properties of the components differ greatly, $\lambda_1/\lambda_2 > 1 \cdot 10^2$, reducing the machine computation time to a fraction of its value, with a corresponding reduction of the accumulated computational error.

The amount of machine time required increases with the ratio λ_1/λ_2. The method considered here was compared with the methods of finite differences [3, 4, 5] and finite elements [6, 7]. The comparison showed that in order to obtain a specified accuracy, the present method requires a smaller number of nodes and less machine time.

NOTATION

λ_1, thermal conductivity of hemisphere; λ_2, thermal conductivity of substance filling the gap between the hemispheres; y, z, coordinate axes; ij, subscripts for coordinates of grid lines; σ_{szij} and R_{szij}, thermal conductance and thermal resistance, respectively, of two-component block ij in the direction of the z axis for isothermal subdivision; ν, ratio of thermal conductivities of components; σ_{syij}, thermal conductance of homogeneous block in the direction of the y axis of the component with thermal conductivity λ_1; σ_{azij}, thermal conductance of two-component block ij in the direction of the z axis for adiabatic subdivision; σ_{ayij}, thermal conductance of homogeneous block in the direction of the y axis of the component with thermal conductivity λ_2; t_{ij}, temperature at the center of block ij; q_{in}, heat flux at inlet of model; q_{out}, heat flux at outlet; λ, effective conductivity of the model; H, height; S, area of base of model; $\bar{\lambda}$, arithmetic mean value of the effective thermal conductivity.

LITERATURE CITED

1. N. Vakao and D. Vortmeyer, Chem. Eng. Sci., 26, 1753 (1971).
2. I. W. Rayleigh, Phil. Mag. (5), 34, 451 (1892).
3. K. J. Binns and P. J. Lawrenson, Analysis and Computation of Electric and Magnetic Field Problems, 2nd ed., Pergamon.

4. A. Tohm and K. Aiplet, Numerical Calculations of Fields in Technology and Physics [Russian translation], Énergiya, Moscow—Leningrad (1964).
5. W. Wasow and J. Forsythe, Difference Methods for the Solution of Partial Differential Equations [Russian translation], IL, Moscow (1963).
6. O. C. Zienkiewicz and J. K. Cheung, "The finite elements in the solution of field problems," Engineer, 200, 507-510 (1965).
7. O. C. Zienkiewicz, "The finite element method: from intuition to generality," Appl. Mech. Rev., No. 3, 249 (1970).

SPECIFIC HEAT OF RHENIUM AT HIGH TEMPERATURES

L. P. Filippov and F. G. Él'darov

UDC 536.212

The results of measurements of the specific heat of a wire sample of rhenium in the temperature interval 1600-2400°K and also data on the electrical resistivity and the integrated degree of blackness are reported.

To date, insufficient attention has been given to the specific heat of rhenium at high temperatures.

Taylor and Finch [1] cited data on the specific heat up to 3100°K, although in fact the experimental material is obtained up to 2000°K, since the electrical resistivity required for the treatment of the corresponding data is measured only up to this temperature and then extrapolated. The results of this extrapolation cannot be regarded as particularly reliable, since it involves the continuation of a curve whose slope is strongly dependent on temperature (it would be more logical to extrapolate the specific heat itself — a weaker function of temperature). The accuracy of the data in Taylor and Finch's paper could also do with being improved upon. In order to find the specific heat in the pulse method of measurement used by these authors, it is necessary to determine the time derivative of a curve produced on the screen of an oscilloscope, a procedure that is associated with a large error.

The measurements reported in [2] are coarse. The data are obtained by photographing the variation in time of the readings of a photoelectric pyrometer on the screen of an oscilloscope; the specific heat is obtained via measurements of the integrated degree of blackness, and the results are presented in the form of a large-scale graph.

The specific heat measurements of rhenium reported in [3] relate to a single crystal sample. They span the temperature interval up to 2500°K, but, as noted previously [3, 4], are of a tentative character, since they were obtained ignoring anisotropy of the single crystal.

In the work reported here the specific heat of wire samples of rhenium was measured using the method previously developed in the Department of Molecular Physics and Mechanics of the Physics Faculty of Moscow State University [5]. This method consists, essentially, in heating the investigated sample by the sum of dc

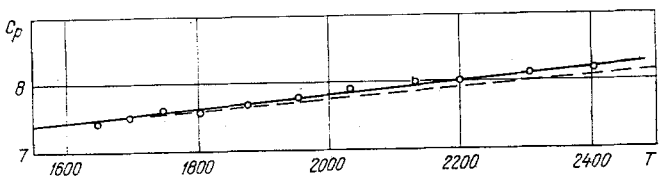

Fig. 1. Measured specific heat of rhenium. The dashed curve was calculated from (5). C_p is in cal/g-atom · °K; temperature T is in °K.

M. V. Lomonosov Moscow State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 2, pp. 292-296, February, 1977. Original article submitted March 17, 1976.